Problem: A ball is tossed **upward** from the Royal Gorge Bridge at a velocity of 6 m/s. If the bridge is 1053 ft above the river, derive **and** solve the differential equation predicting its height as a function of time. Predict the time of impact and assume the balls flight through the air is frictionless. Solve the problem by hand and by using Mathcad's Runge-Kutta function.

we know height is a function of time: $h = f(t)$

where the acceleration is solely due to gravity $\frac{d^2}{dt^2}h(t) = h'' = g$

where: $g = 9.807 \frac{m}{s^2}$

Solving by hand by separating the variables and integrating (shown to the right):

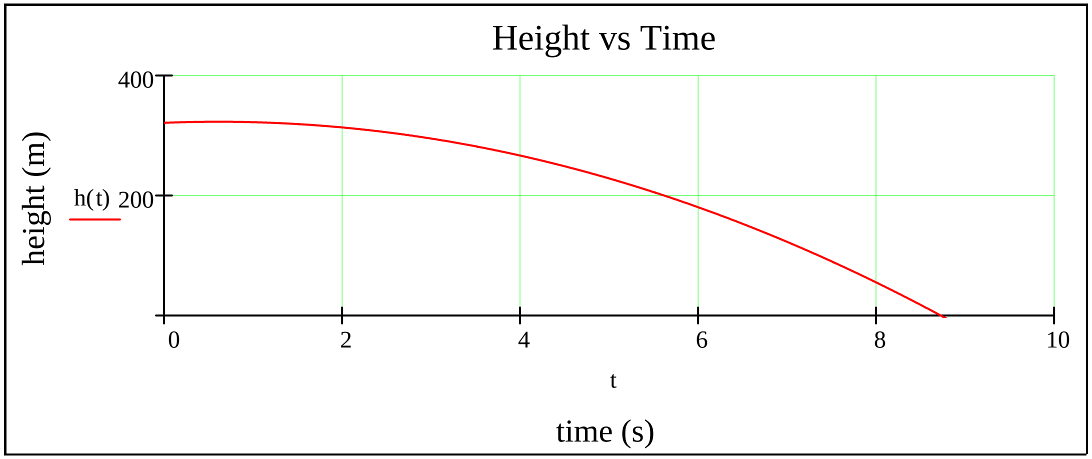
$$h'' = -g \cdot dt^2$$

$$h' = -g \cdot t + c_1$$

$$h(t) = \frac{-g \cdot t^2}{2} + c_1 \cdot t + c_2$$

Establishing a coordinate axis system where downward is negative and upward is positive makes g negative. Any initial vertical velocity is just c_1 while any initial displacement is c_2 (both constants must be signed properly depending upon the initial conditions). The river is established as zero height. This yields the classic ballistics equation ignoring friction.

Inputting the initial conditions: $v_{oy} := 6 \cdot \frac{m}{s}$ $h_o := 1053 \cdot ft$ $h(t) := \frac{-g \cdot t^2}{2} + v_{oy} \cdot t + h_o$



Using Mathcads root finding function to locate time of impact:

initial guess: $t := 8 \cdot s$

$\text{root}(h(t), t) = 8.725\ s$

Impact at 8.725 seconds

Using Runge-Kutta to solve this 2nd order. linear, homogeneous differential equation: $\frac{d^2}{dt^2}h = h'' = g$ or $h'' + 0 \cdot h' - g = 0$

redefining g so it's dimensionless and can be used in Mathcad's rkfixed function: $g := 9.807$

specifying the initial conditions:

initial displacement $h(0) = 1053 \cdot ft$

initial velocity $h''(0) = 6 \cdot \frac{m}{s}$

in an array named **init** $init := \begin{pmatrix} 320.954 \\ 6 \end{pmatrix}$ (meters) (meters/sec upward)

defining an array D showing the relationship of the derivatives (this works for higher order equations also).

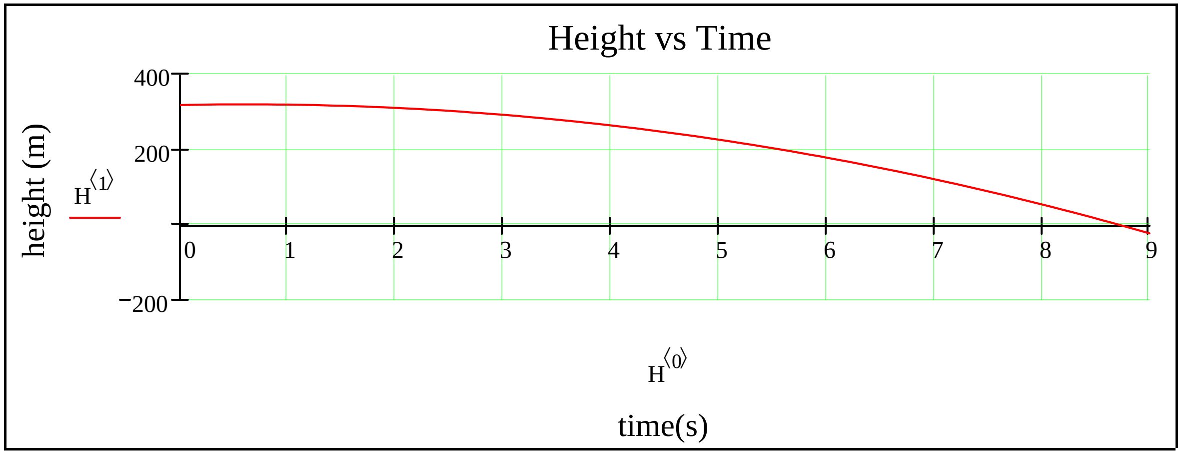
$$D(t, h) := \begin{bmatrix} h_1 \\ 0 \cdot (h_1) + -g \end{bmatrix}$$

define the first derivative

the 2nd derivative (defined in terms of the 1st derivative)

specifying the interval where the solution is to be found using evaluated at 100 points: $start := 0$ $end := 9$ $npoints := 100$

defining the height (H) with the Mathcad function: $H := rkfixed(init, start, end, npoints, D)$



Impact in about 8.7 seconds