FJSHS - Algebra 2

Project 1 - The Cord Problem

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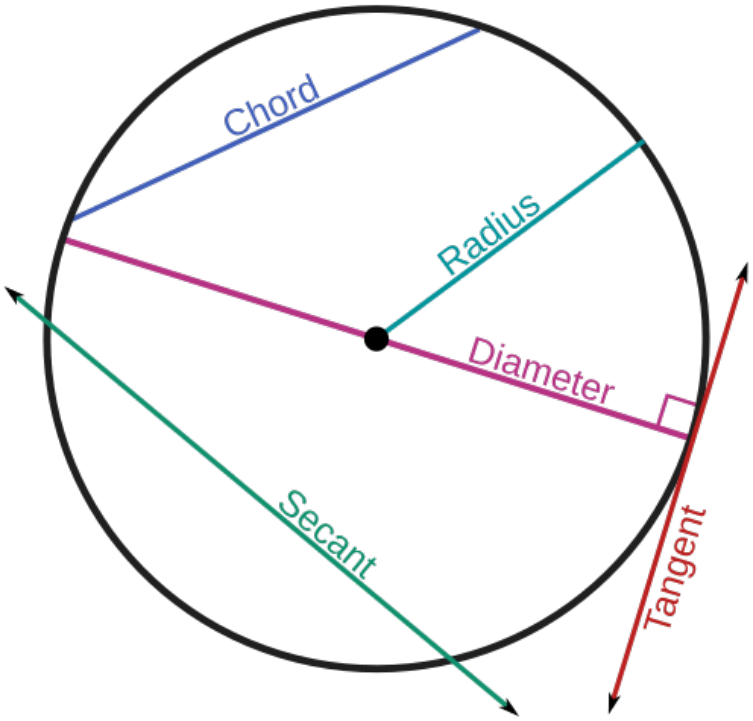
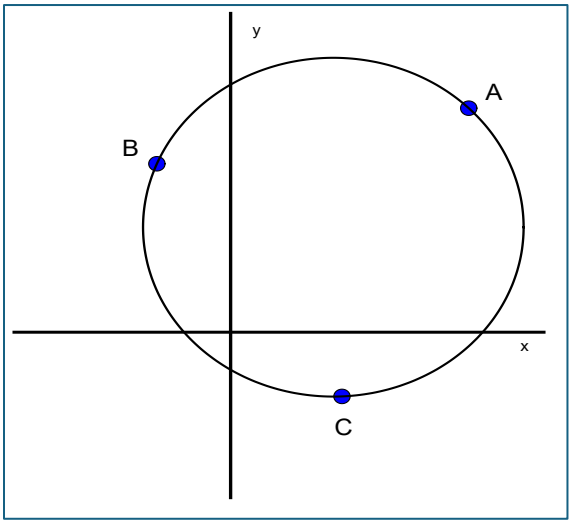


Outline

I. Problem Statement
II. Solution Technique
III. Problem Solution
IV. Check Solution
V. Problem Application

Students please note: I have used slightly different coordinates for the points. You use the points given in the Problem Statement (see Appendix A).



I. Problem Statement:

- Given three points defining a circle:

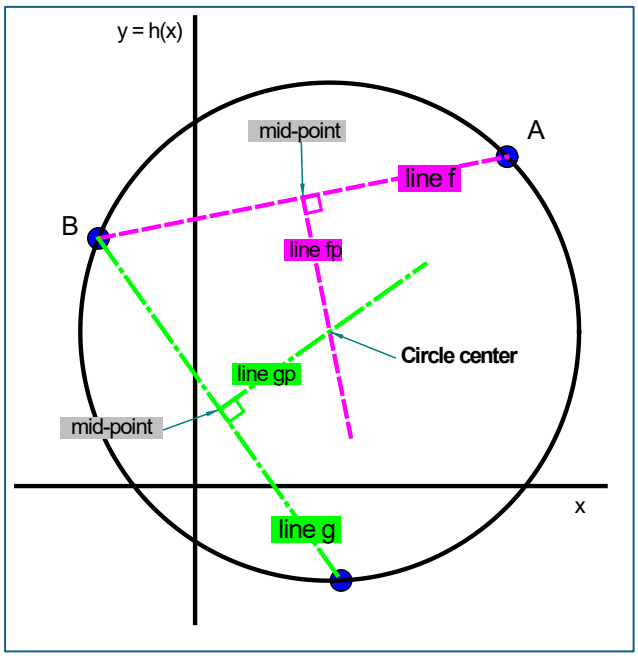
  A (10, 6) B (-2, 7) C (3, -2).

- Calculate the circle center to two decimal places.

II. Solution Technique:

A. From Google using the search words:*"find the center of a circle given three points"* yields: To find the center of a circle from three points, form two chords connecting pairs of points and then construct the perpendicular bisectors of those chords. The point where these two perpendicular bisectors intersect is the center of the circle.

Step-by-step Method



1. **Label the points:**
   Let the three given points be A, B, and C.
2. **Form chords:**
   Connect points A and B with a line segment, points B and C with another line segment.
3. **Find the midpoints:**
   Determine the midpoint of the segment AB the midpoint of the segment BC.
4. **Calculate the slopes:**
   Find the slope of the line segment AB and slope of the line segment BC.
5. **Find the perpendicular bisectors:**
   - The slope of the perpendicular bisector negative reciprocal of the chord's slope.
   - Use the midpoint formula and the perpendicular slope to write the equation of the perpendicular bisector AB. Do the same for the segment BC.
6. **Find the intersection point:**
   Solve the two equations of the perpendicular bisectors simultaneously to find the point of intersection. This point is the center of the circle.
7. **Find the radius:**
   The distance from the center to any of the three points (A, B, or C) is the radius of the circle.

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B. The solution process will follow these steps.

III. Problem Solution

1. **Label the points: The problem provides the three points A, B, and C**
   A (10, 6), B (-2, 7), C (3, -2).

2. **Form the chords:**

- Connect points A and B with a line segment labeled function f. Calculate the function f:

Function f slope: $m_{ab} := \left[\frac{(6)-(7)}{(10)-(-2)}\right] = -0.083$

Function f y-axis intercept: $b_{ab} := 6 - m_{ab}\cdot 10 = 6.833$

Function f equation in slope-intercept form: $f(x) := -0.083\cdot(x) + 6.833$

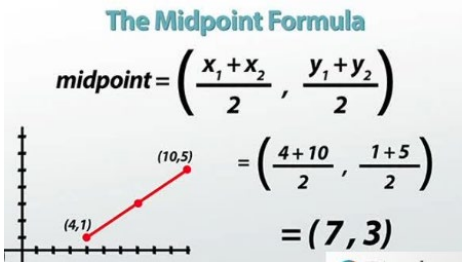
- Connect points B and C with a line segment labeled function g. Calculate the function g:

Function g slope: $m_{bc} := \left[\frac{-2-7}{3-(-2)}\right] = -1.8$

Function g y-axis intercept: $b_{bc} := (-2) - (-1.8)\cdot(3) = 3.4$

Function g equation in slope-intercept form: $g(x) := -1.8\cdot(x) + 3.4$

3. **Find the midpoints:**



- **Function f mid-point:**

$x_{fmp} := \frac{10+(-2)}{2} = 4$ $y_{fmp} := \frac{6+7}{2} = 6.5$ $f_{mp} := (x_{fmp}\ \ y_{fmp}) = (4\ \ 6.5)$

- **Function g mid-point**

$x_{gmp} := \frac{3+(-2)}{2} = 0.5$ $y_{gmp} := \frac{-2+7}{2} = 2.5$ $g_{mp} := (x_{gmp}\ \ y_{gmp}) = (0.5\ \ 2.5)$

4. And 5. Calculate the slopes, y-axis intercepts, equations for perpendicular bisectors f_p and g_p:

Slope of f_p: $m_{abp} := \frac{-1}{-0.083} = 12$

y-axis intercept for f_p $b_{fp} := 6.5 - 12 \cdot 4 = -41.5$

Function f_p equation: $f_p(x) := 12 \cdot (x) - 41.5$

Slope of g_p: $m_{bcp} := \frac{-1}{-1.8} = 0.556$

y-axis intercept for g_p $b_{gp} := 2.5 - (0.556) \cdot (0.5) = 2.222$

Function g_p equation: $g_p(x) := 0.556 \cdot x + 2.222$

5. Find the intersection point of the 2 f_p and g_p:

Solve the two equations of the perpendicular bisectors simultaneously (using substitution) to find the point of intersection. **This point is the center of the circle.**

Solving using substitution:: $f_p(x) = g_p(x)$

$12 \cdot (x) - 41.5 = 0.556 \cdot x + 2.222$

$x := 3.82$

Back substitution: $y := 0.556 \cdot x + 2.222 = 4.35$

ANSWER: Center of the circle = (3.82, 4.35)

6. Find the radius:

(The distance from the center to any of the three points (A, B, or C) is the radius of the circle.)

USING POINT A (10, 6):

The standard equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle, r is its radius, and (x, y) are the coordinates of any point on the circle's circumference. So…

Calculate the radius of the circle: $radius := \sqrt{(x - A_{0,0})^2 + (y - A_{0,1})^2} = 6.398$

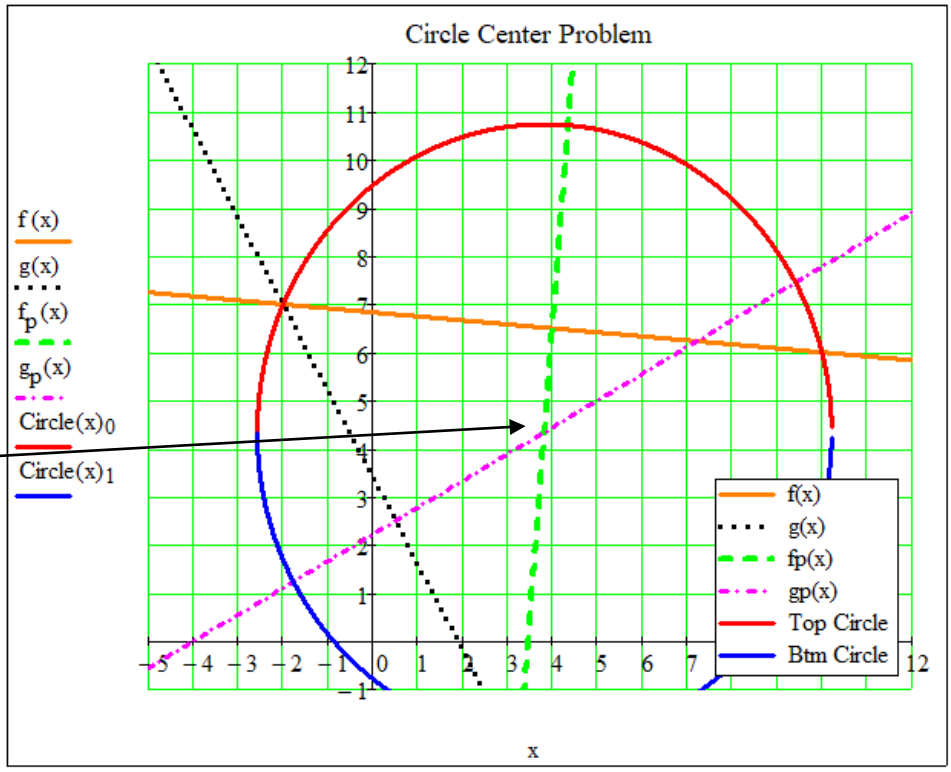
$radius := \sqrt{(3.82 - 10)^2 + (4.35 - 6)^2} = 6.396$

Radius = 6.40

IV. Check Solution by graphing (use Desmos?) the circle as two separate functions (top and bottom).

All equations confirm the answer when accurately graphed.

ANSWER:
Center of the circle = (3.82, 4.35)

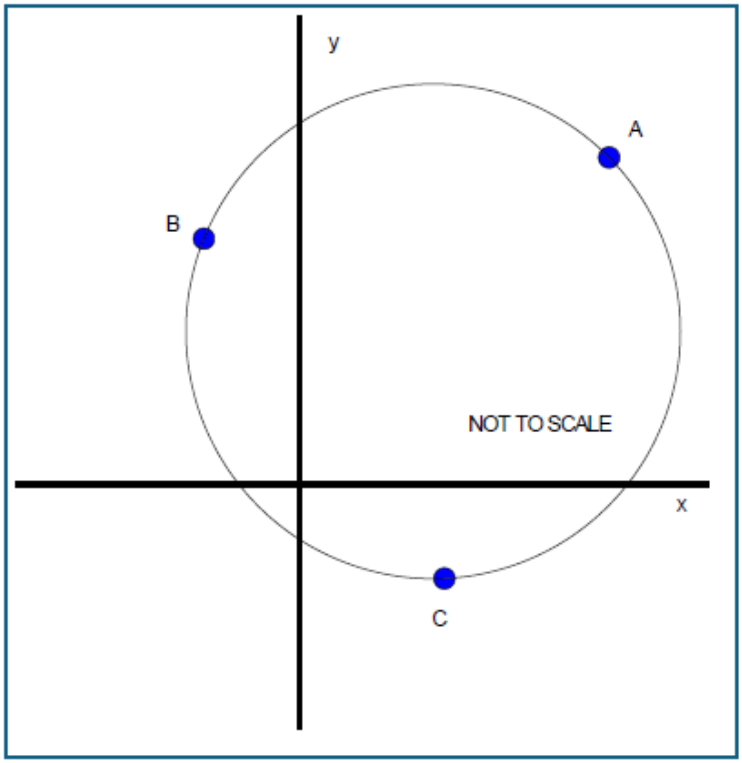


V. Problem Application - You think this out!

Appendix A – Clean copy of the problem statement:

FJSHS – Algebra 2 Project 1 – Cords (50 Points) Name: _____

Given three points A(10.5, 9.6) and B(-2.1, 7.1) and C(4.3, -2.6). Since three point define a circle, find the center of the circle to 2 decimal places. Show all work and explain your thought process. Refer to **Geometry and Algebra 1** references as necessary. Note the image is NOT TO SCALE.



Present your solution in a Professionally formatted MS Word or Google Doc using the following format:

- Section1 – Problem Description.
- Section 2 – Problem solution process /steps.
- Section 3 – Steps solving the problem
- Section 4 – Explicity identify the solution
- Section 5 – Propose a real-world application for the application of this problem.
- Attach appendices as appropriate. Appenicies may contain images of hand calculations and **must** include a clean copy of this problem statement.

Submit this document electronically as an email attachment to mheinen@re-2.org on or before mid-night, Thursday, 9-18-2025. This project may be submitted as a hard copy in lieu of an electronic copy. A hard copy of the project should be retained in student notebooks/binders under a tab labeled "Projects".

Additional copies of this project may be downloaded from Mr. Heinen's website (www.markeredwards.com) under the Algebra 2 tab/page (near the bottom) as a PDF file.